Paper Reference(s)

6680

Edexcel GCE

Mechanics M4

Advanced/Advanced Subsidiary

Wednesday 25 June 2003 – Morning

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Answer Book (AB16)	Nil
Graph Paper (ASG2)	
Mathematical Formulae (Lilac)	

Candidates may use any calculator EXCEPT those with the facility for symbolic algebra, differentiation and/or integration. Thus candidates may NOT use calculators such as the Texas Instruments TI 89, TI 92, Casio CFX 9970G, Hewlett Packard HP 48G.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M4), the paper reference (6680), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
This paper has six questions.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

1. A wooden ball of mass 0.01 kg falls vertically into a pond of water. The speed of the ball as it enters the water is 10 m s^{-1}. When the ball is x metres below the surface of the water and moving downwards with speed v m s^{-1}, the water provides a resistance of magnitude $0.02v^2$ N and an upward buoyancy force of magnitude 0.158 N.

(*a*) Show that, while the ball is moving downwards,

$$-2v^2 - 6 = v\frac{dv}{dx}.$$

(3)

(*b*) Hence find, to 3 significant figures, the greatest distance below the surface of the water reached by the ball.

(5)

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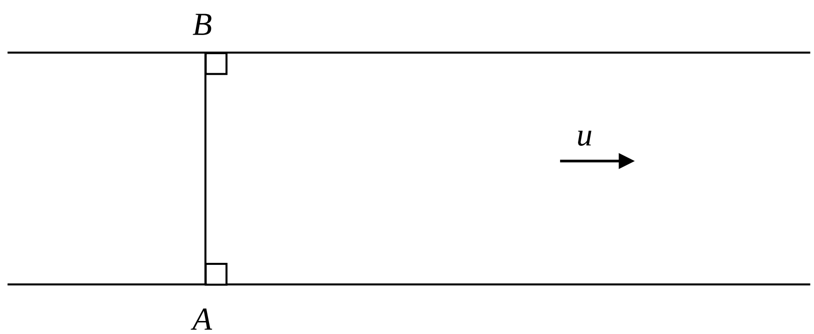
2. **Figure 1**



A man, who rows at a speed v through still water, rows across a river which flows at a speed u. The man sets off from the point A on one bank and wishes to land at the point B on the opposite bank, where AB is perpendicular to both banks, as shown in Fig. 1.

(*a*) Show that, for this to be possible, $v > u$.

(3)

Given that $v < u$ and that he rows from A so as to reach a point C, on the opposite bank, which is as close to B as possible,

(*b*) find, in terms of u and v, the ratio of BC to the width of the river.

(5)

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3. A man walks due north at a constant speed u and the wind seems to him to be blowing *from* the direction 30° east of north. On his return journey, when he is walking at the same speed u due south, the wind seems to him to be blowing *from* the direction 30° south of east. Assuming that the velocity, $\mathbf{w}$, of the wind relative to the earth is constant, find

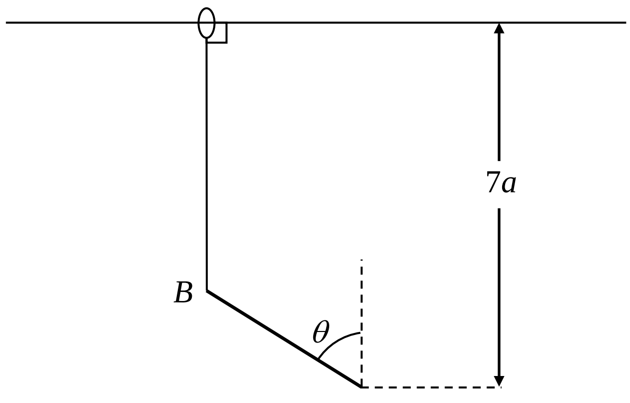
(i) the magnitude of $\mathbf{w}$, in terms of u,

(ii) the direction of $\mathbf{w}$.

(9)

4.

Figure 2



A uniform rod AB, of length $2a$ and mass $8m$, is free to rotate in a vertical plane about a fixed smooth horizontal axis through A. One end of a light elastic string, of natural length a and modulus of elasticity $\frac{4}{5}mg$, is fixed to B. The other end of the string is attached to a small ring which is free to slide on a smooth straight horizontal wire which is fixed in the same vertical plane as AB at a height $7a$ vertically above A. The rod AB makes an angle θ with the upward vertical at A, as shown in Fig. 2.

(*a*) Show that the potential energy V of the system is given by

$$V = \tfrac{8}{5}mg\,a\,(\cos^2\theta - \cos\theta) + \text{constant}.$$

(6)

(*b*) Hence find the values of θ, $0 \leq \theta \leq \pi$, for which the system is in equilibrium.

(5)

(*c*) Determine the nature of these positions of equilibrium.

(4)

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5. A light elastic string, of natural length $2a$ and modulus of elasticity mg, has a particle P of mass m attached to its mid-point. One end of the string is attached to a fixed point A and the other end is attached to a fixed point B which is at a distance $4a$ vertically below A.

(*a*) Show that P hangs in equilibrium at the point E where $AE = \frac{5}{2}a$.

(5)

The particle P is held at a distance $3a$ vertically below A and is released from rest at time $t = 0$. When the speed of the particle is v, there is a resistance to motion of magnitude $2mkv$, where $k = \sqrt{\left(\frac{g}{a}\right)}$. At time t the particle is at a distance $(\frac{5}{2}a + x)$ from A.

(*b*) Show that

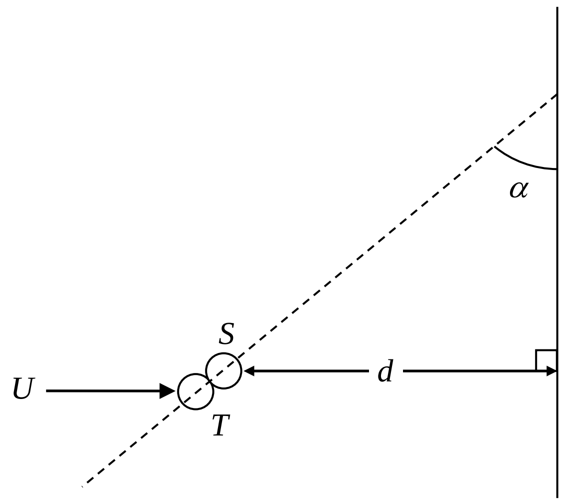
$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2k\frac{\mathrm{d}x}{\mathrm{d}t} + 2k^2x = 0.$$

(5)

(*c*) Hence find x in terms of t.

(7)

6. **Figure 3**



A small smooth uniform sphere S is at rest on a smooth horizontal floor at a distance d from a straight vertical wall. An identical sphere T is projected along the floor with speed U towards S and in a direction which is perpendicular to the wall. At the instant when T strikes S the line joining their centres makes an angle α with the wall, as shown in Fig. 3.

Each sphere is modelled as having negligible diameter in comparison with d. The coefficient of restitution between the spheres is e.

(*a*) Show that the components of the velocity of T after the impact, parallel and perpendicular to the line of centres, are $\frac{1}{2}U(1 - e)\sin\alpha$ and $U\cos\alpha$ respectively.

(7)

(*b*) Show that the components of the velocity of T after the impact, parallel and perpendicular to the wall, are $\frac{1}{2}U(1 + e)\cos\alpha\sin\alpha$ and $\frac{1}{2}U[2 - (1 + e)\sin^2\alpha]$ respectively.

(6)

The spheres S and T strike the wall at the points A and B respectively.

Given that $e = \frac{2}{3}$ and $\tan\alpha = \frac{3}{4}$,

(*c*) find, in terms of d, the distance AB.

(5)

END